Many image quality metrics, including the MTF (Infante, 1985; O'Callaghan & Veron, 1987), the associated MTFA (Snyder, 1973), and the SQRI metric (Barten, 1987), take advantage of the predictable power of linear systems analysis by assuming that CRT displays can be characterized by a single point-spread function. The point-spread function and its Fourier counterpart, the modulation transfer function (MTF) completely characterize linear display systems. The predictive power of linear systems analysis simplifies the description of images displayed on linear display systems. To the extent that CRT pixels can be predicted by a single spatial point-spread function, one can describe any displayed image by the convolution of the image with the display pixel point-spread function (see Naiman & Farrell, 1988).

In this paper we review the important properties of a linear system and describe how to test the linearity of CRT displays. We use the results of our tests of several CRT displays to place confidence limits around our ability to characterize displayed images.

**Background and Terminology**

In order to characterize an image displayed on a particular device we need to know transfer functions of the device. In the case of a CRT display, the relevant transfer functions are the gamma function and the point spread function of CRT pixels. The gamma function describes the mapping between the image values sent to the display controller that determines the intensities of the phosphor gun(s) and the luminous intensity values as recorded by a photometer through a photopic sensor. The point spread function (or psf) of a pixel is the two dimensional luminance distribution of the pixel. If the CRT display is a linear system, the pixel gamma functions and the point spread functions can be used to completely describe the effects of any input that can be imaged on the display.

For example, let the input to a CRT be described as a matrix of image values. The entries in this matrix contain the values corresponding to the intensity of each pixel in the image. The gamma function of each pixel translates the image values into luminance values. The point spread function of each pixel is used to derive a description of the luminance distribution of each pixel in the displayed image. This is accomplished by multiplying each pixel luminance by the pixel point spread function and summing over the entire two dimensional distribution. This operation is known as the convolution of the input image with the point spread function of the display.

If the display is spatially homogeneous only a single point spread function and a single gamma function are needed to specify the display device transfer function. Although CRT pixels are not spatially homogeneous, it may be possible to retain a single gamma and a single psf that can be scaled for spatial location.

**Results**

**Measurement Apparatus**

The measurement system was based on a Pritchard photometer model 1980A manufactured by Photo Research. The photometer was equipped with a spatial scanner attachment for making one dimensional luminosity measurements under the control of a workstation that handles data collection and stimulus presentation. A slit aperture with a 0.4 by 4 degree aperture was used in the measurements in conjunction with a 10X lens allowing 200 - 300 sample points over the pixel's width.

A horizontal scan of a monochrome pixel is shown in Figure 1. The ordinate represents the horizontal scan distance in hundredths of inches (scans have an arbitrary distance offset). The abscissa is the luminous intensity in can- delas per meter squared. This scan data has been fitted using Infante's sum of gaussians method [Infante 1986] and the result is plotted in Figure 1 as a dotted line. All other plots in this paper are plots of raw data.

![Figure 1. Single pixel with Gaussian fit](image)

**Tests of Display Linearity**

A linear system is one which behaves according to the principles of superposition and shift invariance. The principle of superposition states that the output or response of a system (such as a CRT display) to any compound input is equal to the sum of the responses of each individual component in the compound input. If a CRT display is linear then any image composed of many image points can be described as the sum of the responses of all the individual points in the displayed image.

The principle of superposition can be broken down into two critical properties. The first property, additivity can be represented mathematically as

\[ R(a + b) = R(a) + R(b) \]

where R is the response of a linear system and a and b are two inputs to the system.

Additivity implies that the spectral power and the spatial profile of pixels are independent of one another and, therefore, additive. Pixel independence would dictate that adjacent pixels do not affect each other, whereas gun-phosphor independence is manifested by a lack of interaction in the three color
components within a single pixel (note that this has no meaning for the monochrome CRT case).

Gun-phosphor independence in the Trinitron monitor was easily demonstrated. For several different intensity settings in a single pixel, a one dimensional scan (in the horizontal direction) of the pixel's intensity was measured with the Pritchard scanning photometer. Four separate scans were performed at each intensity setting: with only the red, or green, or blue component of a white pixel turned on, and with all three components turned on simultaneously. The results are shown in Figure 2. The sum of the separate components is represented by a solid line and the composite scan with all three components on (i.e. the white pixel) is shown as a dotted line. The correspondence between the white pixel and the sum of the individual phosphor components indicates that the spatial distribution of each of the phosphors are independent of one another (cf. Austin & Otto, 1987) and therefore additive. Furthermore, Brainard (1987) and Cowan & Rowell (1986) have demonstrated that the spectral power of each of the three phosphors are also independent of one another.

Unfortunately, pixel independence appears to be more difficult to achieve. Figure 3 shows a one-dimensional horizontal scan of three adjacent monochrome pixels (shown as a solid line) together with the prediction of additivity based on the sum of scans of the three individual pixels. The disparity between the two plots illustrates the failure of the additivity assumption for adjacent pixels in the horizontal direction. Figure 4 shows similar plots for five adjacent pixels where every other pixel is on. As we would expect, the interaction between pixels decreases when pixels are separated by more than one pixel.

Figure 5 shows a one-dimensional vertical scan of three adjacent monochrome pixels again with the prediction based on the sum of scans of the three individual pixels shown as a dotted line. Additivity in the vertical direction seems to be a good assumption. A pixel's vertical neighbors had very little interactive affect presumably because bandwidth of electron beam and drive electronics is more demanding in the horizontal direction than the vertical. The failure of pixel independence was large when neighboring pixels were turned on in the horizontal direction and the nonlinearities grew worse with the neighboring pixel's intensity value. Finally, our measurements of the Sony Trinitron indicate that the additivity
assumption of adjacent pixels is not violated in the vertical direction, and departure from this assumption in the horizontal direction is not as bad as in the monochrome display.

The second property derived from the principle of superposition is the property of homogeneity or scale invariance: If an input, \(a\), is scaled by a constant, \(f\), the response to the scaled input, \(fa\), will be equal to the response to the original input, \(a\), scaled by the constant \(f\), i.e.,

\[ R(fa) = fR(a) \]

where \(f\) is a scale factor.

We tested the scale invariance of monochrome and Trinitron pixels by scanning the same pixel on each display at 8 different intensity settings from 100% to 30% of the normalized full scale pixel intensity values in 10% steps. If all the points in the luminance profile of a pixel are amplified by a scale factor then the CRT should display the same luminance profile scaled by a single factor.

Figure 6 shows one-dimensional scans of the same monochrome pixel at 8 different intensity levels. A regression analysis was performed to determine if there is a single scale factor that maps each pixel shape into the 100% intensity pixel. The slope of the linear regression provides the scale factor that will map the points in the luminance profile of a pixel at one intensity to the points of the pixel at another intensity. For both monochrome and Trinitron pixels the scale factors that map the pixel at 100% intensity into the pixels at other intensities are predicted by the gamma function of the display. This result indicates that not only is there a scale factor that will map pixels at different intensities into one another, but that the scale factors can be predicted by the gamma functions of the displays.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure6}
\caption{Scaling a single pixel
Horizontal scan, monochrome monitor}
\end{figure}

It is desirable that CRT pixels be not only scale invariant and independent, but shift invariant as well. Ideally the linear properties of the display should be invariant over time and across the surface of the display. In other words, the point spread and gamma functions of a linear display should be the same regardless of where on the display it is measured and regardless of what day it is measured.

It is a well known fact that the gamma and pixel point spread functions of present-day CRT displays are not spatially shift invariant (Brainard, 1989; Hosokawa, et al, 1987). However, CRT displays can be remarkably stable over time. For example, Brainard (1989) found that the gamma function of a Barco CRT display varies between 3 and 5 percent over a three month period. Moreover the drift in the gamma functions over time could be well characterized by a single scale factor. In other words, the gamma functions may have differed by as much as 5 percent but were linearly related to one another by a single scale factor. When gamma functions are corrected by the appropriate factor, the drift in time was less than one percent. Thus far our spatial scanning measurements suggest that the luminance profile of a pixel is also relatively stable over time.

Although CRT displays do not have the ideal property of spatial shift invariance, they may have the property that the gamma and point spread functions change only by a scale factor that depends on the location of a pixel. For example, Brainard (1989) found that although the gamma of a Barco display differs depending on spatial location, there is a single scale factor that maps each gamma function into the other. It is, therefore, possible to store a single gamma function that can be corrected by a scale factor depending on spatial location. Similarly, it may be possible to correct for the spatial inhomogeneity of pixel point spread functions by discovering the scale factor that maps luminance profiles of pixels at different locations into one another. Alternatively, an analytical expression can be fitted to the luminance profile of a pixel and the parameters of the profile stored for different screen locations (Hosokawa, et al, 1987). Finally, it is also possible to define regions of the display for which gamma functions and pixel point spread functions are locally shift invariant. The measurements we are currently making will resolve this issue.

Impact

If accurate reproduction of multiple luminosity level images is a concern, attention should be paid to the display's capability to display images without modification. In particular, if the display is not a linear system then the displayed image will not be the convolution of the pixel point spread function with the desired image. Furthermore, mathematically derived display metrics, like the MTF which assume the linearity of the display system, are subject to error if the displays are not linear.

The root cause of the adjacent pixel nonlinearities is a slew rate limitation in the display system. This is manifested in isolated pixels that never achieve full intensity. Recall the two scans shown in Figure 3: the solid line being the intensity output of three adjacent pixels; and the dotted line representing the sum of two measurement scans, an on-off-on scan summed with an off-on-off scan. In a linear display system the two scans of Figure 3 would coincide. The actual data of the summed scan indicates that an isolated pixel whose prior adjacent pixel is not on never achieves the full output intensity as compared to a pixel whose prior adjacent pixel is on. This result is consistent with a slew rate limitation causing the isolated pixel to start turning off prior to being fully turned on. This limitation is primarily due to the video amplifier subsystem. In order to trim costs, display system video amplifiers are designed to meet a set of minimum criteria of frequency response, rise time and slew rate.

Raster scan CRTs and stroke written CRTs that have an inadequate slewing rate in their video amplifiers are the displays most susceptible to spatial interactions between adjacent pixels. A restricted slew rate in the display system may not be the only cause of the spatial nonlinearities, however. Although digitally addressed displays such as matrix addressed
flat panel displays can be designed to be immune to this problem, the multiplexing of address lines in matrix addressed displays can cause adjacent pixel crosstalk, as well as a non-linear intensity response. Ongoing measurements will determine the extent of this effect in several flat panel display technologies.

In addition to documenting the departure from the linearity assumption, our results also suggest methods for correcting the non-linearities that exist in the display device. Pixel luminance nonlinearity problems can be resolved using two methods: increasing the performance of display hardware and using compensation techniques. Increasing hardware performance can reduce the nonlinearity to an acceptable level, but it would be advantageous to alleviate the effects without paying the price of the higher quality display hardware. Compensation techniques akin to gamma correction could be used to alter the drive signal level to a pixel taking into account the intensity of the pixel's adjacent horizontal neighboring pixels. This technique is suggested and has not been empirically verified.

Conclusions

The measurements made thus far suggest the following conclusions: Within a color pixel the components are additive and scale invariant. Pixel independence does not hold for adjacent pixels, but pixels separated by one or more inactive pixels will be independent. Scale invariance of point spread function was demonstrated by monochrome and Trinitron pixels. Ongoing measurements will determine whether spatial inhomogeneity may be characterized by a single scale factor.

References


